$(1+i) = (1+\pi)(1+i_{real})$

real = 4.878 %

1.) An n-year $1,000 par value bond with 4.20% annual coupons is purchased at a price to yield an annual effective rate of i. You are given:

i. If the annual coupon rate had been 5.25% instead of 4.20%, the price of the bond would have increased by $100.

ii. At the time of the purchase, the present value of all the coupon payments is equal to the present value of the bond's redemption value of $1,000.

Calculate i.

2.) Joe Millionaire deposits X in an account today in order to fund his retirement. He would like to receive payments of $93,600 per year, in real terms, at the end of each year for a total of 25 years, with the first payment occurring 5 years from now.

The inflation rate will be 0.0% for the first four years and 2.5% per annum thereafter.

The annual effective rate of return is 7.5%.

Calculate X.

3.) Ronald takes out a loan to be repaid with annual payments of $500 at the end of each year for 2n years. The annual effective interest rate is 4.94%.

The sum of the interest paid in year 1 plus the interest paid in year n+1 is equal to 720.

Calculate the amount of interest paid in year 10.

4.) Tanner purchases a 100,000 home. Mortgage payments are to be made monthly for 30 years, with the first payment to be made one month from now. The annual effective rate of interest is 5%. After 10 years the amount of each monthly payment is increased by 325.40 in order to repay the mortgage more quickly. Calculate the amount of interest paid over the duration of the loan.

5.) A company agrees to repay a loan over five years. Interest payments are made annually and a sinking fund is built up with five equal annual payments made at the end of each year. Interest on the sinking fund is compounded annually. You are given:

- The amount in the sinking fund immediately after the first payment is X.
- The amount in the sinking fund immediately after the second payment is Y.
- The ratio Y/X = 2.09
- The net amount of the loan immediately after the fourth payment is $3,007.87

Calculate the amount of the sinking fund payment.

6.) You are given the following information about an investment account:

	Jan. 1, 1997	Mar. 1, 1997	T, 1997	Sep. 1, 1997	Jan.1, 1998
Account Value (Before deposit or withdrawal)	\$100	\$108	\$102	\$118	\$130
Deposit			20	9	
Withdrawal		12			

The dollar-weighted yield rate is 12.04% and the time-weighted yield rate is 13.61%.

Calculate T (to the nearest beginning of a month) using the simple interest approximation method.

7.) An insurance company owns a \$1000 par value 10% bond with semi-annual coupons. The bond will mature for \$1000 at the end of 10 years. The company decides to sell the bond. The current yield in the bond market is 7% compounded semi-annually. Two thirds of the proceeds from the sale are invested into a 21-week T-bill and the remainder is used to buy a 5-yr GIC. You are also given the following.

- The discount yield on the T-bill is 5%
- The GIC pays \$510 at maturity at an annual interest rate of j
- y is the effective annual return earned on the T-bill
- $z = j + y$

Calculate z.

8.) The following table gives the pattern of investment year and portfolio interest rates over a three-year period, where $m = 2$ is the time after which the portfolio method is applicable.

Calendar Year of Original Investment y	Investment Year Rates i_1^y	Investment Year Rates i_2^y	Portfolio Rates i^{y+2}	Calendar Year of Portfolio Rate y + 2
z	4.00%	5.50%	10.00%	z + 2
z + 1	6.00%	7.00%	10.00%	
z + 2	6.50%	8.00%		

An investment of \$15,000 is made at the beginning of each of calendar years z, z + 1, and z + 2. What is the average annual effective time-weighted rate of return for the three-year period?

Hard

9.) An annuity-due is purchased at a price of $8000. The annuity makes semi-annual payments for 4 years. The first payment is $400 and each subsequent payment increases by $200.

The payments are reinvested in a fund which earns an annual effective rate i.

Interest payments are received every 6 months and reinvested at a nominal rate of 8%, convertible semi-annually.

The overall effective annual yield on this investment over the 4-year period is 6.994%.

Calculate i.

10.) You are repaying a loan of $10,000 by establishing a sinking fund and making twenty equal payments at the end of each year. The sinking fund earns 7% effective annually.

Immediately after the fifth payment, the yield on the sinking fund increases to 8% effective annually. At that time, you then adjust the sinking fund payment to X so that the sinking fund will still accumulate to $10,000 as originally scheduled.

~~Determine X.~~

9.

	$400i$	$1000i$	$1800i$	$2800i$	$4000i$	$5400i$	$7000i$	$8800i$
400	600	800	1000	1200	1400	1600	1800	
0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4

$8000\,(1.06994)^4 = 10484.02 = 8800 + \text{other}$

$1684.02 = \text{other stuff}$

$400i(1.04)^7 + 1000i(1.04)^6 + 1800i(1.04)^5 + \dots\dots + 8800i$

$= i\left(400(1.04)^7 \dots\dots 8800\right) = i(33677.367) = 1684.02$

Test 3 Solutions

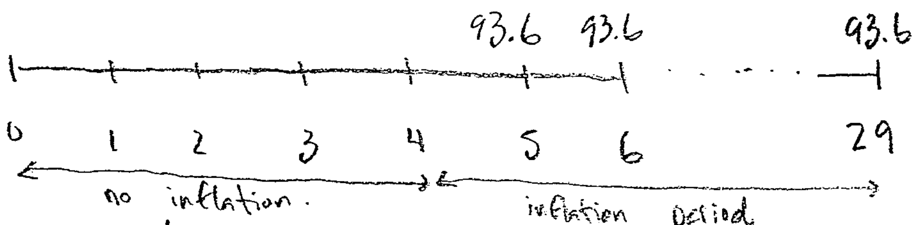
(1) $1000(.0525) = 52.50 \qquad 1000(.042) = 42$

i) $\Delta P = (52.50 - 42) \cdot a_{\overline{n}|i} = 100 \quad \Rightarrow \quad a_{\overline{n}|i} = \dfrac{100}{10.50}$

ii) $42 \cdot a_{\overline{n}|i} = 1000v^n \quad \Rightarrow \quad 42 \left(\dfrac{100}{10.50}\right) = 1000v^n \quad \Rightarrow \quad v^n = .4$

$$\Rightarrow \quad a_{\overline{n}|i} = \frac{100}{10.50} = \frac{(1 - v^n)}{i} = \frac{100}{10.50} = \frac{.6}{i} \quad \Rightarrow i = 6.3\%$$

(2)



(these are real CFs)

$\therefore$ need to discount with i_{real}

$$i_{real} = \frac{1+i}{1+\pi} - 1 = \frac{1.075}{1.025} - 1 = 4.87805\%$$

$$PV_4 = 93.6 \cdot a_{\overline{25}|.0487805} = 1335.469428 \qquad PV_0 = \frac{PV_4}{1.075^4} = 1000$$

or $1M dollars

) I_1 = Interest Portion of 1st pymt $= (L)\cdot i = \left(500\cdot\frac{(1-v^{2n})}{(.0494)}\right)(.0494)$

$\therefore$ P_1 = Principal portion of 1st pymt $= 500-(L)\cdot i = 500 - 500(1-v^{2n})$

$= 500(1-1(1-v^{2n})) = 500v^{2n}$

$P_{n+1} = P_1\cdot(1+i)^n = 500v^{2n}(1+i)^n = 500v^n$

$\therefore$ $I_{n+1} = 500 - 500v^n$

so $I_1 + I_{n+1} = 720 \Rightarrow 500-500v^{2n} + 500 - 500v^n = 720$

$500v^{2n} + 500v^n - 280 = 0$ using quadratic $v^n = .4$

$v^n = .4 = \left(\frac{1}{1.0494}\right)^n \Rightarrow n = 19,\ 2n = 38$

$I_{10} = B_9\cdot i = 500\cdot a_{\overline{38-9}|i}\cdot i = 7621.39\cdot(.0494) = \underline{\underline{\$376.50}}$

) Original Payment $\Rightarrow$ $100000 = P\cdot a_{\overline{30\cdot 12}|\frac{i^{12}}{12}}$ $P = 530.06$

$\frac{i^{(12)}}{12} = (1+i)^{(1/12)} - 1 = .004074124$

New pymt $= 530.06 + 325.40 = 855.46$

aining pymts at time 10 years $= 530.06\cdot a_{\overline{20\cdot 12}|\frac{i^{12}}{12}} = \underline{81068.47}$

new pymt loan will be paid off in n months: $81068.47 = 855.46\cdot a_{\overline{n}|\frac{i^{12}}{12}}$

$\therefore n = 120$

t of interest paid = (total pymts) − original loan amount

$= ((10\cdot 12)\cdot 530.06 + 120(855.46)) - 100000 = \underline{\underline{66261.25}}$

5.) • pymt = X • amount in fund after 2nd pymt $\Rightarrow X(1+j) + X = Y$

$$\frac{Y}{X} = 2.09 \qquad Y = 2.09X \qquad \Rightarrow \qquad X(1+j) + X = 2.09X$$

$$X(1+j) = 1.09X \qquad \therefore \quad 1.09 = 1+j$$

$$j = .09$$

Net amount of the loan after 4th pymt $= L - X \cdot s_{\overline{4}|.09} = 3007.8$

$$= X \cdot s_{\overline{5}|.09} - X \cdot s_{\overline{4}|.09} = 3007.87 = X \cdot (1.09)^4 \qquad X = \underline{\underline{2130.85}}$$

6.) time weighted yield $\Rightarrow \dfrac{108}{100} \times \dfrac{102}{108-12} \times \dfrac{118}{102+X} \times \dfrac{130}{118+9} = 1.1361$

$$102 + X = 121.999 \quad \therefore X = \$20$$

dollar weighted yield $\Rightarrow 100(1+i) - 12\left(1+\frac{(12-2)}{12}\cdot i\right) + 20\left(1+t\cdot i\right) + 9\left(1+\frac{(12-8}{12}\right.$

$= 130$ at $i = 12.04\% \rightarrow \quad 128.1972 + 20t \cdot (.1204) = 130$

$$t = .74867$$

t in months $= 12(.7487) = 8.984$, $\therefore$ the \$20 deposit was invested for ≈ 9 months

which means it was deposited on Apr. 1, 1997. $\underline{\underline{T = \text{Apr. } 1, 1997}}$

7.) Selling Price of bond:

Coupon $= 1000\left(\frac{.10}{2}\right) = \$50$ $\qquad$ $PV = \text{price} = 50 \cdot a_{\overline{20}|.035} + 1000v^{20} = 1213.186$

$\frac{i}{2} = \frac{.07}{2} = 3.5\%$ $\qquad$ so $\quad 1213.186\left(\frac{2}{3}\right) = 808.79$ is invested in a T-bill

$1213.186\left(\frac{1}{3}\right) = 404.3953$ is invested in 5-yr GIC.

GIC $\Rightarrow$ $404.3953(1+j)^5 = 510$ $\quad \therefore \quad j = 4.75\%$

T-bill $\Rightarrow$ $\text{Price} = 808.79 = (\text{Face amount})\left[1 - (.05)\left(\frac{21.7}{360}\right)\right] \Rightarrow \text{Face amount} = 82$

$$y = \left(\frac{\text{Face amount}}{\text{Price}}\right)^{\frac{365}{n}} - 1 = \left(\frac{825.647}{808.79}\right)^{\frac{365}{21.7}} - 1 = .052553$$

$$z = j + y = .0475 + .0525 = .10 \text{ or } 10\%$$

(8) Value of the fund at the end of year 2: $15000(1.04) = 15600$

Value at the end of year 2+1 $= 15600(1.055) + 15000(1.06) = 32358$

Value at the end of year 2+2 $= 15600(1.055)(1.10) + 15000(1.06)(1.07) +$

$15000(1.065) = 51091.80$

$\therefore$ time weighted return $= \left(\frac{15600}{15000}\right) \times \left(\frac{32358}{30600}\right) \times \left(\frac{51091.80}{47358}\right) - 1 = .186455445$

The average annual rate $= (1.186455)^{1/3} - 1 = .05864$ or 5.86%

9. $(1+i)^{1/2} - 1 = j$

pcl	400	600	800	1000	1200	1400	1600	1800	
interest pymt		400j	1000j	1800j	2800j	4000j	5400j	7000j	8800j
	0	1	2	3	4	5	6	7	8

Sum of principal pymts $= 400 + 600 + 800 + \dots 1600 + 1800 = 8800$

FV of interest pymts at $\left(\frac{.08}{2}\right)$ per 6 months $= j\left(400(1.04)^7 + 1000(1.04)^6 + 1800(1.04)^5 + \dots\right.$

$\left. 7000(1.04) + 8800\right) =$

$j(33677.367)$

since 4 year return is 6.994% we get,

$$8000(1.06994)^4 = 8800 + j(33677.367)$$

$$\frac{10484.0162 - 8800}{33677.367} = j = .05$$

$(1+j)^2 - 1 = i$

$1.05^2 - 1 = .1025$

or $i = 10.25$

10. original pymt $\Rightarrow$ $P \cdot s_{\overline{20}|.07} = 10\,000$

$P = 243.93$

Value in fund after 5th pymt $= 243.93 \cdot s_{\overline{5}|.07} = 1402.77$

$$FV = 10\,000 = 1402.77(1.08)^{15} + X \cdot s_{\overline{15}|.08}$$

$$= 4449.835 + X \cdot 27.1521 \qquad \therefore X = 204.41$$